Reliability Block Method
Reliability models

- **based on success**
  - ✓ Reliability Block Diagram (RBD)
  - ✓ Markov models
  - ✓ Petri nets graph
  - ✓ Event Tree diagram

- **based on failure**
  - ✗ FMEA (Failure Mode and Effects Analysis)
  - ✗ FMECA (Failure Mode, Effects and Criticality Analysis) / AMDEC (Analyse des modes de défaillance, de leurs effets et de leur criticité)
  - ✗ FTD (Fault Tree Diagram)
  - ✗ Cause-Consequence Diagram
A **reliability block diagram** (RBD) is a diagrammatic method for showing how component reliability contributes to the success or failure of a complex system. RBD is also known as a **dependence diagram** (DD).

**Simplifying assumptions:**

1. the system and its components have **two states**: functional and inoperative
2. the failures are independent

**Models:**

- Serial
- Parallel
- Mixed (decomposable to basic serial or parallel structures)
- Non-decomposable
Serial system reliability block diagram

To ensure functionality of a serial system, all of its blocks (items) must successfully function during the intended mission time of the system.

E$_i$ – “i element is functional” event

S – “system is functional” event

S = E$_1$ \(\cap\) E$_2$ \(\cap\) ... \(\cap\) E$_n$

\[ P(S) = P(E_1 \cap E_2 \cap ... \cap E_n) \]

\[ P(S) = \prod_{i=1}^{n} P(E_i) \]

\[ R_S(t) = \prod_{i=1}^{n} R_i(t) \]

\[ F_S(t) = 1 - \prod_{i=1}^{n} (1 - F_i(t)) \]
Parallel system reliability block diagram

For functional success of a parallel (or redundant) system, at least one of its blocks (items) must successfully function during the intended mission time of the system (failure is caused only if all items are inoperative).

\[ S = E_1 \cup E_2 \cup ... \cup E_n \]

\[ \overline{S} = \overline{E_1} \cup \overline{E_2} \cup ... \cup \overline{E_n} = \overline{E_1} \cap \overline{E_2} \cap ... \cap \overline{E_n} \] (De Morgan)

\[ P(\overline{S}) = P(\overline{E_1} \cap \overline{E_2} \cap ... \cap \overline{E_n}) \]

\[ P(\overline{S}) = \prod_{i=1}^{n} P(\overline{E_i}) \]

\[ F_S(t) = \prod_{i=1}^{n} F_i(t) \]

\[ R_S(t) = 1 - \prod_{i=1}^{n} (1 - R_i(t)) \]
Mixed system reliability block diagram

In mixed configuration, the elements are connected in serial and parallel topology to perform a required system operation. In this case, to compute system reliability, the network is broken into serial or parallel subsystems. The reliability of each subsystem is determined and then the system reliability may be obtained on the basis of the relationship between the subsystems.
Non-decomposable system reliability block diagram

- specific for complex systems (e.g.: telecommunication networks)
- the network is broken into serial or parallel subsystems
- contains special network topology: star, delta, ring, mesh.

Reliability evaluation techniques:
- Bayes’ Theorem (conditional probability)
- Minimal Path Set Method
- Minimal Cut Set Method
- Exhaustive enumeration of discrete states
- Delta – Star Technique
Conditional probability theorem (Bayes)

• $A_1, A_2, ..., A_n$ – events that form a complete system
• $S$ – an event that belongs to this system

$$P(S) = \sum_{i=1}^{n} P(S | A_i) \cdot P(A_i)$$

• In reliability theory, a complete system can have events with two states: functional and inoperative for a critical element

$$P(S) = P(A_i) \cdot P(S | A_i) + P(\bar{A}_i) \cdot P(S | \bar{A}_i),$$ where $A_i$ - critical element

$$P(S) \rightarrow R_S$$

$$P(A_i) \rightarrow R_{A_i}$$

$$P(\bar{A}_i) \rightarrow (1 - R_{A_i})$$

• This can be iterated upon multiple critical elements
Minimal Path Set Method

- **Path set** – the subset of functional elements which ensures system functionality, regardless of the state of all other elements. Example: $\text{ABED}$

- **Minimal path set** – the path set with the *least* amount of functional elements which allows system functionality. Example: $\text{AB}$

- In order for the system to **function**, $L_1 \text{ OR } L_2 \text{ OR } \ldots \text{ OR } L_m$ must function (parallel topology).

- In order for the path set to **function**, all of the elements must be functional (serial topology).

$$S = L_1 \cup L_2 \cup \ldots \cup L_m$$

$$P(S) = P(AB \cup CD \cup AED \cup CEB)$$
Minimal Cut Set Method

- **Cut Set** – the subset of elements that, when inoperative, render the system inoperative regardless of the state of all other elements. Example: **ACEB**
- **Minimal Cut Set** – the cut set which contains no subset of elements whose failure alone render the system inoperative. Example: **AC**

In order for the system to **fail**, $T_1 \lor T_2 \lor \ldots \lor T_k$ must be inoperative (serial topology).

In order for a cut set to be inoperative, all of its elements must be inoperative. (parallel topology).

$$\overline{S} = T_1 \cup T_2 \cup \ldots \cup T_k$$

$$P(\overline{S}) = P(\overline{AC} \cup \overline{BD} \cup \overline{AED} \cup \overline{CEB})$$
Exhaustive enumeration of discrete states

- identifying all possible states of the given system ($2^n$ states)
- sum the states that lead to $S=1$ by replacing “1” with $R_i$ and “0” with $F_i$

$$R_S = F_A F_B R_C R_D R_E + F_A F_B R_C R_D R_E + F_A R_B R_C F_D R_E + ... + R_A R_B R_C R_D F_E + R_A R_B R_C R_D R_E \text{ (16 terms)}$$